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Two-stage stochastic operation framework for optimal management of the water–energy–hub

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Abstract: This study proposes a bi-level stochastic framework to address optimal scheduling of energy hub (EH) in a pool-based short-term market considering electrical–thermal–water demands. EH acts as an independent price-maker producer in a day-ahead electricity market aiming to maximise its profit. The market settlement mechanism is constructed as the pay-at-market-clearing price (MCP), where each producer/consumer is paid at the MCP. The problem model is formulated as a bi-level optimisation approach in a stochastic environment, in which the upper level defines the profit maximisation of the proposed strategic producer, whereas the lower-level expresses the dispatch cost of the considered power grid. This results in a problem formulation with mathematical equilibrium constraints which is transformed into a new mixed-integer linear programme based on Karush–Kuhn–Tucker conditions. A stochastic framework based on unscented transform is developed to model the high uncertainties of EH water demand, EH thermal demand, EH electric demand, generators and loads submitted price to the market. The simulation results on the IEEE test system advocate the effectiveness and appropriate performance of the proposed strategic EH producer in the electricity market and its effect on the locational marginal prices of buses in a transmission-constrained market.

Nomenclature

Sets/indices

i, j	bus indicators
Ω^{GB}/g	set/index of generation units
Ω^{EHB}/eh	set/index of EH
Ω^{DLB}/D	set/index of demand loads
Ω^{Br}/br	set/index of lines
Ω^{T}/t	set/index of periods
Ω^{w}/w	set/index of scenarios
Ω^{κ}/κ	set/index of uncertain variables
$\Omega^{D_{eh}}/D_{eh}$	set/index of EH demand

Constants

α_{e}^{loss}	loss efficiency of EH battery
$\alpha_{g,w,t}$, $\beta_{D,w,t}$	bidding price of generation units and demand loads during time slot t in scenario w
CF^{Des}	energy consumption of the desalination unit
M	big M number
$\eta^{G2H-Boi}$, η_{ee}^{T}, η_{e}^{ch}, η_{e}^{dch}, $\eta^{G2E-CHP}$, $\eta^{G2H-CHP}$	efficiency of the gas-to-heat conversion of the boiler, transformer electricity, battery charging, battery discharging, gas-to-electricity conversion of the CHP and gas-to-heat conversion of the CHP, respectively
$\underline{P}_{eh}$, $\bar{P}_{eh}$, $\underline{S}^{re}$, $\bar{S}^{re}$	minimum/maximum EH output power/battery charger level
$\underline{P}^{S}$, $\bar{P}^{S}$	minimum/maximum battery charging rate
$\bar{P}_{br}$	maximum allowable power injection through the lines
$P_{D_{eh},w,t}$	EH demand during time slot t in scenario w
prob_{w}	probability of scenario w
$\bar{P}^{B}/\bar{P}^{T}/\bar{P}^{CHP}$	maximum capacity of the boiler/transformer/CHP
$\underline{P}_{g,w,t}$, $\bar{P}_{g,w,t}$, $\underline{P}_{D,w,t}$, $\bar{P}_{D,w,t}$	minimum/maximum generator/demand power
$\bar{S}^{re}/\underline{S}^{re}$	maximum/minimum remaining charge of the battery storage
$\underline{V}^{ST}$, $\bar{V}^{ST}$	minimum/maximum volume of secondary tank
$\bar{V}^{DT}$	maximum volume of desalination tank
$\underline{W}^{ID}$, $\bar{W}^{ID}$	minimum/maximum input water of desalination tank
$\bar{W}^{OD}$	maximum output water of desalination unit
$\boldsymbol{Y}_{ij}$	admittance imaginary part matrix

Problem variables

$\gamma_{eh,w,t}$	EH bidding price during time slot t in scenario w
$\mathrm{LMP}_{eh,w,t}$	locational marginal price (LMP) of the related EH bus (\$/MWh)
$\mu_{g,w,t}^{1,2}$, $\mu_{D,w,t}^{1,2}$, $\mu_{eh,w,t}^{1}$, $\mu_{ij,w,t}^{1,2}$	Lagrangian function multipliers related to generation units, demand loads, EH and lines i–j
$P_{g,w,t}$	generator output power during time slot t in scenario w
$P_{D,w,t}$	power demand during time slot t in scenario w
$P_{eh,w,t}$	EH power generation during time slot t in scenario w
$P_{w,t,br}$	power injected through the lines during time slot t in scenario w (MW)

$P^{\text{ch}}_{\text{w, t}}$, $P^{\text{dch}}_{\text{w, t}}$	charging and discharging powers of the battery during time slot *t* in scenario *w*
$P^{\text{GICHP}}_{\text{w, t}}$	combined heat and power (CHP) input gas power during time slot *t* in scenario *w*
$P^{\text{TH}}_{\text{w, t}}$	EH thermal demand during time slot *t* in scenario *w*
$P^{\text{GIB}}_{\text{w, t}}$	boiler input gas power during time slot *t* in scenario *w*
$P^{\text{GIN}}_{\text{w, t}}$	EH gas input power during time slot *t* in scenario *w*
$P^{\text{Des}}_{\text{w, t}}$	desalination unit electrical power consumed during time slot *t* in scenario *w*
$S^{\text{re}}_{\text{w, t}}$	battery remaining energy during time slot *t* in scenario *w*
$\theta_{i,\text{ w, t}}$	voltage angle of the *i*th bus during time slot *t* in scenario *w*
$V^{\text{ST}}_{\text{w, t}}$	secondary tank water volume during time slot *t* in scenario *w*
$V^{\text{DT}}_{\text{w, t}}$	desalination unit water volume during time slot *t* in scenario *w*
$W^{\text{OD}}_{\text{w, t}}$	desalination unit output water during time slot *t* in scenario *w*
$W^{\text{out}}_{\text{w, t}}$	secondary tank output water during time slot *t* in scenario *w*
$W^{\text{ID}}_{\text{w, t}}$	desalination unit input water during time slot *t* in scenario *w*

Binary variables

$I^{\text{ch}}_{\text{w, t}}$, $I^{\text{dch}}_{\text{w, t}}$	binary variable indicating the charging and discharging modes
$I^{\text{D}}_{\text{w, t}}$	desalination unit operational binary variable
$m^{1,2}_{ij,\text{ w, t}}$	linearisation binary variable related to the line *i*–*j*
$m^{1,2}_{\text{g, w, t}}$	linearisation binary variable related to generation units
$m^{1,2}_{\text{D, w, t}}$	linearisation binary variable related to adjustable loads
$m^{1}_{\text{eh, w, t}}$	linearisation binary variable related to the EH

1 Introduction

Significant expansion of power grids, along with acute global energy growth and the growing need and use of the energy, are some substantial motivations for developing some systems so-called energy hub (EH). The EH is a multi-carrier energy system with several inputs and outputs with different conversion systems based on gas and electrical infrastructures, encompassing thermal–electrical consumptions of residential, commercial and industrial loads in an efficient manner. The use of such systems would be a great innovation from the perspective of energy and environmental issues in power systems [1–3]. In [4, 5], the focus is on the expansion planning of the EH. In their investigations, they have used linear energy flows formulation for simplicity aiming to create long-term planning horizon for the EH systems. They have also assessed the EH system reliability indices along with the planning horizon. Moreover, in [5], Dolatabadi *et al.* addressed the design of EH integrated with wind generations. However, due to the inherent uncertainty of the output generated power of the wind turbines, the long-term planning among such energy sources is still a matter of debate. Recently, the advent of communication apparatuses and large quantity of different types of energies (electrical, thermal, gas and water) demands has led the researchers to propose efficient solutions against such crises. In [6], the authors tried to investigate the operation problem of the EH in the smart grid. They have provided a smart platform for the system for minimising the operation cost. A scenario-based approach investigated to model the uncertainty of electricity, gas price and demands. They have also provided a risk assessment based on conditional value at risk (CVaR) technique. Reducing the operation cost has been aimed in [7] by considering the thermal market for participation of EH. They also utilised the scenario-based technique for modelling the uncertainty parameters. In [8], the residential EH operation is investigated in the smart grid wherein the systems are equipped with communication systems. They also provided different mathematical models for household appliances. Demand response has been mentioned by Dolatabadi and Mohammadi-Ivatloosome [9] and Pazouki and Haghifam [10] for mitigating the operation cost. In addition, Similar to the [6], Dolatabadi and Mohammadi-Ivatloo [9] provided the wind-integrated EH CVaR-based scheduling in a smart environment. Pazouki and Haghifam [10] assumed that the EH water demand is directly supplied by the network. This would no longer be available for poor water regions or inaccessible areas to the water network. Also, they have not properly addressed proper scheduling over the water energy for the EH demands. The problem with the operation of such systems, if properly addressed, leads to power loss reduction [11] and emission reduction [12], in such a way that the cost/benefit objectives are considered.

The emergence and development of electricity markets have provided market participants in a competitive environment with the opportunity to generate power. Any market participant in a pool-based electricity market, where all generation/consumption units will participate with their energy and/or price bids, can be a price-maker which is able to alter the market price to its own profit [13] or price-taker which seeks to buy/sell its hourly power, regardless of the market price [14, 15]. In [15], Conejo *et al.* proposed a bidding strategy of a producer for self-scheduling its units aiming to participate in the pool market. Zugno *et al.* [16] and Dai and Qiao [17] investigated the bidding/offering strategies of wind power producers (WPPs). They both proposed bi-level optimisation problem, in which the WPPs can participate in both day-ahead and balancing markets. Unlike [17] where the WPPs act as price-maker permanently, Zugno *et al.* [16] assumed the WPPs act as a price-taker in day-ahead market and price-maker in balancing market. A scenario-based stochastic optimal bidding strategy of a price-maker electric vehicle aggregator is represented in [18]. An approach based on load forecasting for bidding strategy is proposed in [19] for a retailer with flexible demands. Mohsenian-Rad [20, 21] has reconsidered the energy storage system for storing energy and participate in day-ahead market. In [21], the strategy is based on bi-level optimisation process. However, considering the large capacity of the energy storages installed in the network, the operation procedure is not optimally analysed since the main problem does not contain the battery storages degradation cost minimisation as well as the optimal places of the storages. Many references tried to investigate bidding/offering strategies based on a robust optimisation method [22, 23], risk-constrained method [24, 25] and risk-averse method [26, 27]. Avatefipour and Nafisian [28] proposed a new combined method based on clonal selection algorithm (CSA) and artificial neural network (ANN) for the short-term load forecasting application. The proposed technique exploits both the ANN's learning properties for solving the non-linear and complex problems and CSA population-based algorithm for global and local searches.

This paper is most comparable with the studies in [21, 29]. The scenario-based approach for uncertainty modelling is not a proper method since it is a computationally complex and high time-consuming process and is not recommended for correlation modelling among WPPs in a certain place. Besides, to the best of our knowledge, no prior work expresses the market participation of an EH as a price-maker unit. Hence, this paper is dedicated to the operation of a strategic independently operated EH in a pool-based day-ahead electricity market. The market mechanism is pay-at-market-clearing price (MCP), in which the buyers/sellers will be paid at the MCP. The proposed stochastic optimisation problem has been formulated as a mathematical programme with equilibrium constraints (MPECs), in which scenarios are generated based on the unscented transform (UT) method [30]. Owing to the high uncertainty effects, a stochastic framework based on UT is developed to model the uncertainties associated with the EH water demand, EH electricity demand, EH heat demand, generators bidding price in the market and loads submitted price to the market. The problem is inherently non-linear and has been transformed into a mixed-integer linear programme (MILP) using Karush–Kuhn–Tucker (KKT) conditions, which will be more specifically explained in the next sections. The distinguished contributions of this paper lie in the following aspects:

i. This paper proposes an effective model for the market participation mechanism of a strategic price-maker EH in a day-ahead transmission-constrained electricity market. The proposed EH is assumed to be investor owned and seeks to maximise its own profit.
ii. An effective water energy scheduling is proposed for the EH water demand.
iii. Considering different operating cases from a distribution grid point of view and discussing the applicability of the proposed EH participation for every case in detail.
iv. An efficient stochastic optimisation model based on UT is developed to model the uncertainty effects in the proposed two-stage problem. Using such method, the uncertainty associated with the EH water demand, EH electricity demand, EH heat demand, generators bidding price in the market and load submitted price to the market are modelled and imposed to the problem. It should be noted that the proposed EH uses an economic bidding method.
v. Investigating the congestion issue of the network power lines in the presence of the EH.

The remainder of this paper is organised as follows: the problem formulation is provided in Section 2. Section 3 represents MILP formation of the main problem. Section 4 describes the UT method. Results are provided in Section 5, and finally, the work is concluded in Section 6.

2 Mathematical formulation

The EH is a multi-carrier energy system which is made up of multiple storages, grids, network technologies and control systems. The special features of this technology make it a unique equipment for varied applications ranging from the small homes to the large buildings, microgrids and even distribution systems. In this paper, the EH considered is a small size case suitable for a microgrid. Moreover, it can attend the energy market for increasing the EH profit and reducing its operation costs. Detailed representation of problem variables, objective functions and constraints of the problem are provided as follows.

2.1 Mathematical modelling

2.2 Objective functions

The proposed stochastic optimisation problem framework, as a pool-based market participation procedure, can be formulated as follows:

- *Upper level: maximisation of the EH profit:* This objective function guarantees the profit maximisation of the EH through the day-ahead market participation

$$\max : f_1(\mathbf{X}) = \sum_{w \in \Omega^{\mathrm{w}}} \sum_{t \in \Omega^{\mathrm{T}}} \sum_{\mathrm{eh} \in \Omega^{\mathrm{EHB}}} (\mathrm{LMP}_{\mathrm{eh,\,w,\,t}} \times P_{\mathrm{eh,\,w,\,t}}) \times \mathrm{prob}_{\mathrm{w}} \tag{1}$$

- *Lower level: dispatch cost minimisation:* Initially, in order to determine the locational marginal prices (LMPs) of the power grid buses and the generation/consumption power of each unit, the following economic dispatch (ED) will be performed by the independent system operator (ISO) which needs to be minimised:

$$\begin{aligned} \min : f_2(X) = & \sum_{g \in \Omega^{\mathrm{GB}},\, w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}}} \alpha_{\mathrm{g,\,w,\,t}} \times P_{\mathrm{g,\,w,\,t}} \\ & - \sum_{D \in \Omega^{\mathrm{DLB}},\, w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}}} \beta_{\mathrm{D,\,w,\,t}} \times P_{\mathrm{D,\,w,\,t}} \\ & + \sum_{\mathrm{eh} \in \Omega^{\mathrm{EHB}},\, w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}}} \gamma_{\mathrm{eh,\,w,\,t}} \times P_{\mathrm{eh,\,w,\,t}} \end{aligned} \tag{2}$$

The three parameters α, λ and β in (2) are used as cost coefficients in the three layers of water, electricity and thermal in the EH. Using these parameters, the total cost of EH esteemed from the water demand, electricity demand and thermal demand is calculated.

2.3 Constraints

The constraints related to the proposed problem are detailed separately, as represented below:

(i) The constraints corresponded to the ED problem are as follows: these constraints are related to the lower-level problem.

- *Power balance equality constraint at each bus:* The Kirchhoff's current law imposed in each bus aiming to find the power generation of each unit and power injection through the lines. This constraint satisfies the power–load balance in each bus

$$P_{\mathrm{g,\,w,\,t}} - P_{\mathrm{D,\,w,\,t}} + P_{\mathrm{eh,\,w,\,t}} = \sum_{w \in W,\, t \in T,\, br \in \Omega^{\mathrm{Br}}} P_{\mathrm{w,\,t,\,br}} \quad \forall g \in \Omega^{\mathrm{G}},\, D \in \Omega^{\mathrm{DLB}},\, \mathrm{eh} \in \Omega^{\mathrm{E}} \tag{3}$$

- *Active power generation limits of the generators:* This constraint limits the power generation of each unit in each scenario, during time duration t. This constraint is considered due to the technical limitations

$$\underline{P}_{g,\,w,\,t} \le P_{g,\,w,\,t} \le \bar{P}_{g,\,w,\,t}, \quad \forall g \in \Omega^{G},\, w \in \Omega^{w},\, t \in \Omega^{T} \tag{4}$$

- *Active power consumption limits of the demands:* This constraint limits the power demand of each dispatchable load demand in each scenario, during time duration t

$$\underline{P}_{D,\,w,\,t} \le P_{D,\,w,\,t} \le \bar{P}_{D,\,w,\,t}, \quad \forall D \in \Omega^{\mathrm{DLB}},\, w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}} \tag{5}$$

- *Active power generation/consumption limits of the EH:* This constraint limits the active power generation/consumption of the EH in each scenario, during time duration t

$$\underline{P}_{eh,\,w,\,t} \le P_{eh,\,w,\,t} \le \bar{P}_{eh,\,w,\,t}, \quad \forall eh \in \Omega^{\mathrm{EHB}},\, w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}} \tag{6}$$

- *Power flow limits of the branches:* This constraint will keep the power injection through each transmission line away being deviated from its nominal capacity in each scenario during time t. This is caused due to the thermal limits of the feeders

$$|P_{\mathrm{w,\,t,\,br}}| \le \bar{P}_{\mathrm{br}}, \quad \forall \mathrm{br} \in \Omega^{\mathrm{br}},\, w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}} \tag{7}$$

(ii) The constraints corresponded to the EH maximisation problem can be represented as follows: these constraints are related to the upper level.

- *EH's battery is not allowed to be charged/discharged simultaneously:* This constraint will allow the EH's battery to get charged or discharged in each scenario during time t. This constraint determines the charging or discharging status of the battery

$$0 \le I^{\mathrm{ch}}_{\mathrm{w,\,t}} + I^{\mathrm{dch}}_{\mathrm{w,\,t}} \le 1, \quad \forall w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}} \tag{8}$$

- *Available battery charge limit:* This constraint limits the available energy stored in the battery, which will not allow the charging/discharging process being deviated from the nominal capacity of the energy storage

$$\underline{S}^{re} \le S^{\mathrm{re}}_{\mathrm{w,\,t}} \le \bar{S}^{\mathrm{re}}, \quad \forall w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}} \tag{9}$$

- *Time dependency of battery power charge/discharge:* This constraint indicates the battery stored energy in each scenario during each time t. The discharged power of the battery is provided with '−' symbol which will reduce the stored energy of the battery

$$S^{re}_{\mathrm{w,\,t}} = \left(1 - \alpha^{loss}_{e}\right) S^{re}_{\mathrm{w,\,t-1}} + P^{\mathrm{ch}}_{\mathrm{w,\,t}} - P^{\mathrm{dch}}_{\mathrm{w,\,t}}, \quad \forall w \in \Omega^{\mathrm{w}},\, t \in \Omega^{\mathrm{T}} \tag{10}$$

- *Charge limit of the battery:* This constraint limits the charging power of the battery within its maximum charge rate

$$\frac{1}{\eta_e^{ch}}\underline{P}^{S}I^{ch}_{w,t} \le P^{ch}_{w,t} \le \frac{1}{\eta_e^{ch}}\bar{P}^{S}I^{ch}_{w,t}, \quad \forall w \in \Omega^{w}, t \in \Omega^{T} \tag{11}$$

- *Discharge limit of the battery:* This constraint limits the discharging power of the battery within its maximum discharge rate

$$\eta_e^{dch}\underline{P}^{S}I^{dch}_{w,t} \le P^{dch}_{w,t} \le \eta_e^{dch}\bar{P}^{S}I^{dch}_{w,t}, \quad \forall w \in \Omega^{w}, t \in \Omega^{T} \tag{12}$$

- *EH power balance:* The total net power generated and used in the EH should equal as follows:

$$P_{D_{eh},w,t} = \eta_{ee}^{T}P_{eh,w,t} + \eta^{G2E-CHP}P^{GICHP}_{w,t} + P^{dch}_{w,t} - P^{ch}_{w,t} - P \\ \forall D_{eh} \in \Omega^{D_{eh}}, \forall eh \in \Omega^{EHB}, w \in \Omega^{w}, t \in \Omega^{T} \tag{13}$$

- *EH thermal demand:* The amount of thermal power produced and used in the EH should be equal as follows:

$$P^{TH}_{w,t} = \eta^{G2H-CHP}P^{GICHP}_{w,t} + \eta^{G2H-Boi}P^{GIB}_{w,t}, \quad \forall w \in \Omega^{w}, \\ t \in \Omega^{T} \tag{14}$$

- *EH gas input power:* The amount of EH gas input power equals the total boiler input gas and the combined heat and power (CHP) input gas power

$$P^{GIN}_{w,t} = P^{GICHP}_{w,t} + P^{GIB}_{w,t}, \quad \forall w \in \Omega^{w}, t \in \Omega^{T} \tag{15}$$

- *EH energy conversion constraints*

$$\eta_{ee}^{T}P_{eh,w,t} \le \bar{P}^{T}, \quad \forall eh \in \Omega^{EHB}, w \in \Omega^{w}, t \in \Omega^{T} \tag{16a}$$

$$\eta^{G2H-CHP}P^{GICHP}_{w,t} \le \bar{P}^{CHP} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{16b}$$

$$\eta^{G2H-Boi}P^{GIB}_{w,t} \le \bar{P}^{B} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{16c}$$

2.4 Water demand formulation

In this paper, the water demand of the EH has been modelled considering a desalination unit, a primary and a secondary tank. The desalinated water is stored in the primary tank, which is connected to the secondary one. It should be mentioned that the secondary tank has two inputs/outputs. A bilateral flow connection considered between the secondary tank and the water grid, in which the water is able to be injected or consumed to/from the water grid. One input injects water to the secondary tank and one output provides the EH water demand. Fig. 1 shows an illustration of the proposed concept. Details of the water supplement system are shown in Fig. 2.

Below, the water demand formulation is provided, which needs to be added to the EH maximisation problem:

- *Secondary water tank operational constraint*

$$V^{ST}_{w,t} = V^{ST}_{w,t-1} + W^{OD}_{w,t} - W^{out}_{w,t} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{17}$$

- *Secondary water tank operational constraint*

$$\underline{V}^{ST} \le V^{ST}_{w,t} \le \bar{V}^{ST} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{18}$$

- *Primary water tank operational constraint*

$$V^{DT}_{w,t} = V^{DT}_{w,t-1} + W^{ID}_{w,t} - W^{OD}_{w,t} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{19}$$

- *Primary water tank capacity limit*

$$0 \le V^{DT}_{w,t} \le \bar{V}^{DT} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{20}$$

- *Desalination unit input limit*

$$\underline{W}^{ID} \cdot I^{D}_{w,t} \le W^{ID}_{w,t} \le \bar{W}^{ID} \cdot I^{D}_{w,t} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{21}$$

- *Desalination unit output limit*

$$0 \le W^{OD}_{w,t} \le \bar{W}^{OD} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{22}$$

- *Desalination unit consumed power*

$$P^{Des}_{w,t} = W^{ID}_{w,t} \cdot CF^{Des} \quad w \in \Omega^{w}, t \in \Omega^{T} \tag{23}$$

3 Linearisation procedure

By considering (1) and (2), it can be determined that the non-linearity in the above formulation process involves two major facts, which are as follows:

(i) *Definition of the proposed equations:* It can be concluded that the EH's maximised profit value and the LMP of the bus that the EH is connected to is obtained once the market agents' bidding are specified.
(ii) *Multiplication of problem variables:* The problem faces a strategic EH producer, in which the EH optimal price-energy bidding needs to be appropriately evaluated in the proposed optimisation process from the perspective of the ISO.

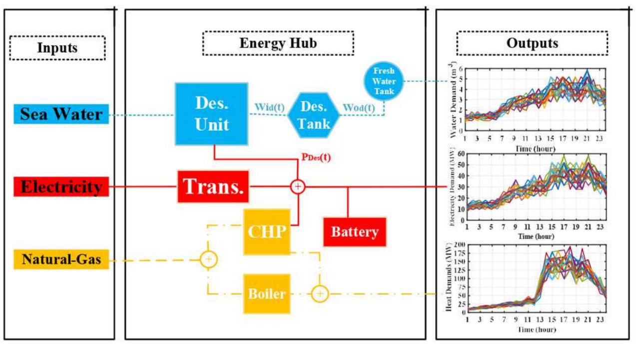

Fig. 1 *Illustration of the proposed EH*

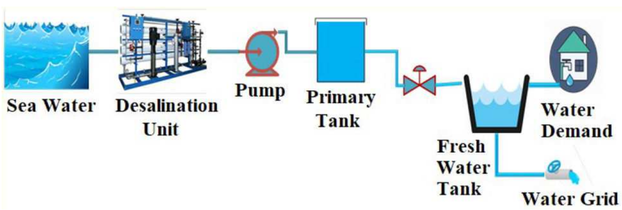

Fig. 2 *Illustration of the water supplement system*

Assuming that f_2(X) is a linear convex problem, by applying KKT conditions to the dispatch cost minimisation problem, we have:

- *Stationary condition*

$$\alpha_{\mathrm{g,w,t}} - \mathrm{LMP}_{\mathrm{eh,w,t}} - \mu^1_{\mathrm{g,w,t}} + \mu^2_{\mathrm{g,w,t}} = 0 \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, g \in \Omega^{\mathrm{GB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{24}$$

$$-\beta_{\mathrm{D,w,t}} + \mathrm{LMP}_{\mathrm{eh,w,t}} - \mu^1_{\mathrm{D,w,t}} + \mu^2_{\mathrm{D,w,t}} = 0 \quad \forall D \in \Omega^{\mathrm{DLB}}, \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{25}$$

$$\gamma_{\mathrm{eh,w,t}} - \mathrm{LMP}_{\mathrm{eh,w,t}} - \mu^1_{\mathrm{eh,w,t}} + \mu^2_{\mathrm{eh,w,t}} = 0 \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{26}$$

$$-\sum_{j>i} \tilde{Y}_{ij}\big(\mu^1_{ij,\mathrm{w,t}} - \mu^2_{ij,\mathrm{w,t}}\big) + \sum_{i>j} \tilde{Y}_{ji}\big(\mu^1_{ji,\mathrm{w,t}} - \mu^2_{ji,\mathrm{w,t}}\big) + \sum_{i \neq j} \tilde{Y}_{ij}\mathrm{LMP}_{ij,\mathrm{w,t}} - \sum_{i \neq j} \tilde{Y}_{ji}\mathrm{LMP}_{ji,\mathrm{w,t}} = 0 \quad \forall (i,j) \in \Omega^{\mathrm{br}}, \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{27}$$

- *Complementary slackness*

$$\mu^1_{ij,\mathrm{w,t}}\big(\bar{P}_{\mathrm{br}} + \tilde{Y}_{ij}(\theta_{i,\mathrm{w,t}} - \theta_{j,\mathrm{w,t}})\big) = 0 \quad \forall (i,j) \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{28}$$

$$\mu^2_{ij,\mathrm{w,t}}\big(\tilde{Y}_{ij}(\theta_{i,\mathrm{w,t}} - \theta_{j,\mathrm{w,t}}) - \bar{P}_{\mathrm{br}}\big) = 0 \quad \forall (i,j) \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{29}$$

$$\mu^1_{\mathrm{g,w,t}}\big(\underline{P}_{\mathrm{g,w,t}} - P_{\mathrm{g,w,t}}\big) = 0 \quad \forall g \in \Omega^{\mathrm{GB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{30}$$

$$\mu^2_{\mathrm{g,w,t}}\big(P_{\mathrm{g,w,t}} - \bar{P}_{\mathrm{g,w,t}}\big) = 0, \quad \forall g \in \Omega^{\mathrm{GB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{31}$$

$$\mu^1_{\mathrm{D,w,t}}\big(\underline{P}_{\mathrm{D,w,t}} - P_{\mathrm{D,w,t}}\big) = 0, \quad \forall D \in \Omega^{\mathrm{DLB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{32}$$

$$\mu^2_{\mathrm{D,w,t}}\big(P_{\mathrm{D,w,t}} - \bar{P}_{\mathrm{D,w,t}}\big) = 0, \quad \forall D \in \Omega^{\mathrm{DLB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{33}$$

$$\mu^1_{\mathrm{eh,w,t}}\big(\underline{P}_{\mathrm{eh,w,t}} - P_{\mathrm{eh,w,t}}\big) = 0, \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{34}$$

$$\mu^2_{\mathrm{eh,w,t}}\big(P_{\mathrm{eh,w,t}} - \bar{P}_{\mathrm{eh,w,t}}\big) = 0, \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{T}} \tag{35}$$

- *Primal feasibility*

$$P_{\mathrm{g,w,t}} - P_{\mathrm{D,w,t}} + P_{\mathrm{eh,w,t}} = \sum_{\mathrm{br}\Omega^{\mathrm{br}}} P_{\mathrm{w,t,br}}, \quad \forall g \in \Omega^{\mathrm{G}}, D \in \Omega^{\mathrm{DLB}}, \mathrm{eh} \in \Omega^{\mathrm{EHB}} \tag{36}$$

$$\underline{P}_{\mathrm{g,w,t}} \le P_{\mathrm{g,w,t}} \le \bar{P}_{\mathrm{g,w,t}}, \quad \forall g \in \Omega^{\mathrm{G}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{37}$$

$$\underline{P}_{\mathrm{D,w,t}} \le P_{\mathrm{D,w,t}} \le \bar{P}_{\mathrm{D,w,t}}, \quad \forall D \in \Omega^{\mathrm{D}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{38}$$

$$\underline{P}_{\mathrm{eh}} \le P_{\mathrm{eh,w,t}} \le \bar{P}_{\mathrm{eh}}, \quad \forall \mathrm{eh} \in \Omega^{\mathrm{eh}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{39}$$

$$|P_{\mathrm{w,t,br}}| \le \bar{P}_{\mathrm{br}}, \quad \forall \mathrm{br} \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{40}$$

- *Dual feasibility*
  (see (41))

Constraints (28)–(35) demonstrate that the above linear formulation still contains non-linear terms. Hence, the proposed constraints are able to be linearised using binary variables. For instance, non-linearity in constraint (28) can be replaced with two constraints utilising an arbitrary binary variable $m^1_{ij,w,\mathrm{t}}$ as follows:

$$\mu^1_{ij,w,\mathrm{t}} = 0; \quad \text{if } m^1_{ij,w,\mathrm{t}} = 1, \quad \forall (i,j) \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{42}$$

$$\bar{P}_{\mathrm{br}} + \tilde{Y}_{ij}\big(\theta_{i,\mathrm{w,t}} - \theta_{j,\mathrm{w,t}}\big) = 0; \quad \text{if } m^1_{ij,\mathrm{w,t}} = 0, \quad \forall (i,j) \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{43}$$

$$\begin{cases} \mu^2_{\mathrm{g,w,t}} \le \big(1 - m^2_{\mathrm{g,w,t}}\big)M \\ \bar{P}_{\mathrm{g,w,t}} - P_{\mathrm{g,w,t}} \le (m^2_{\mathrm{g,w,t}})M \end{cases}, \quad \forall g \in \Omega^{\mathrm{GB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{44), (45}$$

where M is a big number. The same definition is valid for the rest of the non-linear terms, which are represented by the following constraints in (29)–(35):

$$\begin{cases} \mu^2_{ij,\mathrm{w,t}} \le \big(1 - m^2_{ij,\mathrm{w,t}}\big)M \\ \tilde{Y}_{ij}\big(\theta_{i,\mathrm{w,t}} - \theta_{j,\mathrm{w,t}}\big) - \bar{P}_{\mathrm{br}} \le (m^2_{ij,\mathrm{w,t}})M \end{cases}, \quad \forall (i,j) \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{46), (47}$$

$$\begin{cases} \mu^1_{\mathrm{g,w,t}} \le \big(1 - m^1_{\mathrm{g,w,t}}\big)M \\ P_{\mathrm{g,w,t}} - \underline{P}_{\mathrm{g,w,t}} \le (m^1_{\mathrm{g,w,t}})M \end{cases}, \quad \forall g \in \Omega^{\mathrm{GB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{48), (49}$$

$$\begin{cases} \mu^2_{\mathrm{g,w,t}} \le \big(1 - m^2_{\mathrm{g,w,t}}\big)M \\ \bar{P}_{\mathrm{g,w,t}} - P_{\mathrm{g,w,t}} \le (m^2_{\mathrm{g,w,t}})M \end{cases}, \quad \forall g \in \Omega^{\mathrm{GB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{50), (51}$$

$$\begin{cases} \mu^1_{\mathrm{D,w,t}} \le \big(1 - m^1_{\mathrm{D,w,t}}\big)M \\ P_{\mathrm{D,w,t}} - \underline{P}_{\mathrm{D,w,t}} \le (m^1_{\mathrm{D,w,t}})M \end{cases}, \quad \forall D \in \Omega^{\mathrm{DLB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{52), (53}$$

$$\begin{cases} \mu^2_{\mathrm{D,w,t}} \le \big(1 - m^2_{\mathrm{D,w,t}}\big)M \\ \bar{P}_{\mathrm{D,w,t}} - P_{\mathrm{D,w,t}} \le (m^2_{\mathrm{D,w,t}})M \end{cases}, \quad \forall D \in \Omega^{\mathrm{DLB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{54), (55}$$

$$\begin{cases} \mu^1_{\mathrm{eh,w,t}} \le \big(1 - m^1_{\mathrm{eh,w,t}}\big)M \\ P_{\mathrm{eh,w,t}} - \underline{P}_{\mathrm{eh,w,t}} \le (m^1_{\mathrm{eh,w,t}})M \end{cases}, \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{56), (57}$$

$$\begin{cases} \mu^2_{\mathrm{eh,w,t}} \le \big(1 - m^2_{\mathrm{eh,w,t}}\big)M \\ \bar{P}_{\mathrm{eh,w,t}} - P_{\mathrm{eh,w,t}} \le (m^2_{\mathrm{eh,w,t}})M \end{cases}, \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{58), (59}$$

As mentioned before, optimisation problem $f_1(X)$ also contains a non-linear term due to the multiplication of the problem variables. Since $f_1(X)$ is a non-convex non-linear optimisation, it needs to be transformed into easily solved linear equation systems. By considering (26) and multiplying it by $P_{\mathrm{eh,w,t}}$, aiming to reform $f_1(X)$, it becomes the following equation:

$$\gamma_{\mathrm{eh,w,t}} \cdot P_{\mathrm{eh,w,t}} - LMP_{\mathrm{eh,w,t}} \cdot P_{\mathrm{eh,w,t}} - \mu^1_{\mathrm{eh,w,t}} \cdot \underline{P}_{\mathrm{eh,w,t}} + \mu^2_{\mathrm{eh,w,t}} \cdot \bar{P}_{\mathrm{eh,w,t}} = 0, \quad \forall \mathrm{eh} \in \Omega^{\mathrm{EHB}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{60}$$

$$\mu^1_{\mathrm{g,w,t}} \ge 0, \quad \mu^2_{\mathrm{g,w,t}} \ge 0, \quad \mu^1_{\mathrm{D,w,t}} \ge 0, \quad \mu^2_{\mathrm{D,w,t}} \ge 0, \quad \mu^1_{\mathrm{eh,w,t}} \ge 0, \quad \mu^2_{\mathrm{eh,w,t}} \ge 0, \quad \mu^1_{ij,\mathrm{w,t}} \ge 0, \quad \mu^2_{ij,\mathrm{w,t}} \ge 0 \quad \forall g \in \Omega^{\mathrm{GB}}, D \in \Omega^{\mathrm{DLB}}, \mathrm{eh} \in \Omega^{\mathrm{EHB}}, (i,j) \in \Omega^{\mathrm{br}}, w \in \Omega^{\mathrm{w}}, t \in \Omega^{\mathrm{t}} \tag{41}$$

By taking the sum of (60), substituting (33) and (34) into the above equation, and reordering the terms, we attain (see (61)) .

For convex primal $f_2(X)$, one can consider strong duality as follows: (see (62)) .

After putting (61) into (62) and reordering the terms, we have (see (63)) . The non-linear terms have been eliminated in the resulting formulation. Equation (63) is an MILP that can be solved using a CPLEX linear programming solver.

4 Stochastic framework based on UT

The modelling and analysis of high-dimensional dependent uncertainties are of great significance for the reliability and security of future power systems and other engineering domains [31, 32]. UT method is one the powerful and fast tools for modelling the uncertainty. In this paper, an appropriate model needs to be utilised to model the uncertainty of the output generated power of the generators and electric–thermal–water demand load of EH. Assume a non-linear equation $V_{\rm O} = F(\boldsymbol{V}_{\rm I})$, in which $\boldsymbol{V}_{\rm I}$ is the output vector of the stochastic input vector $\boldsymbol{V}_{\rm O}$. Assume p uncertain variables with a normal distribution, mean value and standard deviation of $\bar{m}$ and D, respectively. The UT method will solve the problem $2p + 1$ times to model the uncertainty. The UT method is represented through the following steps:

Step 1: calculating $2p + 1$ samples

By using the following equations, $2r + 1$ samples are calculated:

$$\boldsymbol{V}_I^0 = \bar{m} \tag{64}$$

$$\boldsymbol{V}_{\rm I}^{\kappa} = \bar{m} + \left(\sqrt{\frac{r}{1 - W^0} Q_{V_{\rm I}}}\right)_{\kappa}, \quad \forall \kappa \in \Omega^{\kappa} \tag{65}$$

$$\boldsymbol{V}_{\rm I}^{\kappa + p} = \bar{m} - \left(\sqrt{\frac{r}{1 - W^0} Q_{V_{\rm I}}}\right)_{\kappa}, \quad \forall \kappa \in \Omega^{\kappa} \tag{66}$$

where $\boldsymbol{Q}_{V\rm I}$ denotes the input covariance matrix and also W^0 is the weight corresponding to the $\overline{V_{\rm I}} = \bar{m}$.

Step 2: calculating the weights associated with each sample point

$$\omega^0 = \omega^0 \tag{67}$$

$$\omega^{\kappa} = \frac{1 - \omega^0}{2p}, \quad \forall \kappa \in \Omega^{\kappa}, \kappa = 1, 2, \ldots, p \tag{68}$$

$$\omega^{\kappa + p} = \frac{1 - \omega^0}{2p}, \quad \forall \kappa \in \Omega^{\kappa}, \kappa + p = p + 1, \ldots, 2p \tag{69}$$

Also, the sum of the weights must be 1.

Step 3: calculating the mean output vector $\bar{\boldsymbol{V}}_{\rm O}$ and the output covariance matrix $\boldsymbol{Q}_{V\rm O}$ as follows:

$$\bar{\boldsymbol{V}}_{\rm O} = \sum_{l \in \Omega^l} \omega^l V_{\rm O}^l, \quad \forall l \in \Omega^l, l = 1, 2, \ldots, 2p \tag{70}$$

$$\boldsymbol{Q}_{V_{\rm O}} = \sum_{l \in \Omega^l} W^l (V_{\rm O}^l - \bar{V}_{\rm O})\left(V_{\rm O}^l - \bar{V}_{\rm O}\right)^T, \quad \forall l \in \Omega^l, l = 1, 2, \ldots, 2p \tag{71}$$

The UT model makes use of the first few moments of the probability density function to capture the uncertainty effects with very low computational efforts. Considering a problem of p uncertain parameters, the proposed stochastic framework needs $2p + 1$ concentration points to model the uncertainty effects. A special feature of this stochastic model is the capability of modelling the correlated uncertainty. This helps not only to model the independent uncertainty, but also model the uncertainty effects in the correlated environment. Fig. 3 shows the flowchart of the proposed stochastic method.

5 Results and discussion

The proposed pool-based market participation procedure is evaluated on an IEEE 9-bus test system comprised of three generators, three adjustable loads, the EH and its electrical–thermal–water demand load (see Fig. 4). To illustrate the effects of different strategies on the expected system objective function, four different case studies have been considered to show various aspects of the network performance and will be discussed in details. All simulations were performed in a 24 h time horizon utilising GAMS 24.1.2 and solved using the CPLEX method. Results were obtained using a PC with a 2.8 GHz core and 4 GB of RAM.

The four case studies that have been considered in this work are as follows:

Case I: no congestion lines (base case).
Case II: one congested line (line 5–4).
Case III: one congested line (line 5–6).
Case IV: two congested lines (line 5–6 and 5–4).

In case I, no line exceeds its maximum flow limit, which considered a base case of the problem. In case II, it is assumed that the capacity of line 5–4 is limited and will be congested. The same assumption considered for line 5–6 in case III. In case IV, both lines 5–6 and 5–4 are congested simultaneously. An independently operated EH is assumed to be on bus number 5 in the power grid in all the case studies to supply EH demand load. The EH also participates in the proposed wholesale market as a market agent.

5.1 Assumptions

The following assumptions are made throughout this paper:

i. Proposed transmission network considered to be lossless. Therefore, an optimal DC power flow has been used in this paper.
ii. EH's battery energy storage assumed to be an ideal energy storage unit.
iii. Network demand loads assumed to be adjustable.
iv. Energy storage unit initial energy level is 100 MWh.
v. EH is not able to inject/consume water to/from the water grid at the same time.

5.2 Case I: base case: analysis of the test system with no congested lines

In this case, it has been assumed that all the power line flows are within their desired flow limits. Generators, loads and the EH considered as market agents and submitted their bids in a 24 h daily horizon wholesale market. By implementing the proposed model and after a proper analysis, the optimal values of the problem variables are obtained which is shown in Table 1.

5.3 Case II: analysis of the test system with one congested line (line 5–4)

In this case, the proposed strategic EH in the power grid will supply its demand load and participate in an electricity market as a market agent. Also, line 5–4 flow capacity is limited, which results in a limited flow through the proposed line.

5.4 Case III: analysis of the test system with one congested line (line 5–6)

In this case, it has been assumed that the capacity of line 5–6, which is connected to the EH bus is limited while all the other lines capacities are within their allowable flow limits.

$$\begin{aligned}\sum_{\mathrm{eh}\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} (\mathrm{LMP}_{\mathrm{eh,\,w,\,t}}\cdot P_{\mathrm{eh,\,w,\,t}}) &= \sum_{\mathrm{eh}\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} (\gamma_{\mathrm{eh,\,w,\,t}}\cdot P_{\mathrm{eh,\,w,\,t}}) \\ &- \sum_{\mathrm{eh}\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \left(\mu^1_{\mathrm{eh,\,w,\,t}}\cdot \underline{P}_{\mathrm{eh,\,w,\,t}}\right) + \sum_{\mathrm{eh}\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \left(\mu^2_{\mathrm{eh,\,w,\,t}}\cdot \bar{P}_{\mathrm{eh,\,w,\,t}}\right)\end{aligned} \tag{61}$$

$$\begin{aligned}&\sum_{g\in\Omega^{\mathrm{GB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \alpha_{\mathrm{g,\,w,\,t}}\times P_{\mathrm{g,\,w,\,t}} - \sum_{D\in\Omega^{\mathrm{DLB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \beta_{\mathrm{D,\,w,\,t}}\times P_{\mathrm{D,\,w,\,t}} \\ &+ \sum_{eh\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \gamma_{\mathrm{eh,\,w,\,t}}\times P_{\mathrm{eh,\,w,\,t}} = \sum_{g\in\Omega^{\mathrm{GB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{g,\,w,\,t}}\times \underline{P}_{i,\,\mathrm{w,\,t}} \\ &- \sum_{g\in\Omega^{\mathrm{GB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{g,\,w,\,t}}\times \bar{P}_{\mathrm{g,\,w,\,t}} + \sum_{D\in\Omega^{\mathrm{DLB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{D,\,w,\,t}}\times \underline{P}_{\mathrm{D,\,w,\,t}} \\ &- \sum_{D\in\Omega^{\mathrm{DLB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{D,\,w,\,t}}\times \bar{P}_{\mathrm{D,\,w,\,t}} \\ &+ \sum_{\mathrm{eh}\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{eh,\,w,\,t}}\times \underline{P}_{\mathrm{eh}} - \sum_{\mathrm{eh}\in\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{eh,\,w,\,t}}\times \bar{P}_{\mathrm{eh}} \\ &- \sum_{\mathrm{br}\in\Omega^{\mathrm{br}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{br,\,w,\,t}}\times \bar{P}^{\mathrm{br}}_{\mathrm{w,\,t}} - \sum_{\mathrm{br}\in\Omega^{\mathrm{br}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{br,\,w,\,t}}\times \bar{P}^{\mathrm{br}}_{\mathrm{w,\,t}}\end{aligned} \tag{62}$$

$$\begin{aligned}&\sum_{\Omega^{\mathrm{EHB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mathrm{LMP}_{\mathrm{eh,\,w,\,t}}\cdot P_{\mathrm{eh,\,w,\,t}} = -\sum_{g\in\Omega^{\mathrm{GB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \alpha_{\mathrm{g,\,w,\,t}}\times P_{\mathrm{g,\,w,\,t}} \\ &+ \sum_{D\in\Omega^{\mathrm{DLB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \beta_{\mathrm{D,\,w,\,t}}\times P_{\mathrm{D,\,w,\,t}} + \sum_{g\in\Omega^{\mathrm{GB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{g,\,w,\,t}}\times \underline{P}_{\mathrm{g,\,w,\,t}} \\ &- \sum_{g\in\Omega^{\mathrm{GB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{g,\,w,\,t}}\times \bar{P}_{\mathrm{g,\,w,\,t}} + \sum_{D\in\Omega^{\mathrm{DLB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{D,\,w,\,t}}\times \underline{P}_{\mathrm{D,\,w,\,t}} \\ &- \sum_{D\in\Omega^{\mathrm{DLB}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{D,\,w,\,t}}\times \bar{P}_{\mathrm{D,\,w,\,t}} - \sum_{Br\in\Omega^{\mathrm{br}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^1_{\mathrm{Br,\,w,\,t}}\times \bar{P}^{\mathrm{br}}_{\mathrm{w,\,t}} \\ &- \sum_{\mathrm{Br}\in\Omega^{\mathrm{br}},\, w\in\Omega^{\mathrm{w}},\, t\in\Omega^{\mathrm{T}}} \mu^2_{\mathrm{Br,\,w,\,t}}\times \bar{P}^{\mathrm{br}}_{\mathrm{w,\,t}}\end{aligned} \tag{63}$$

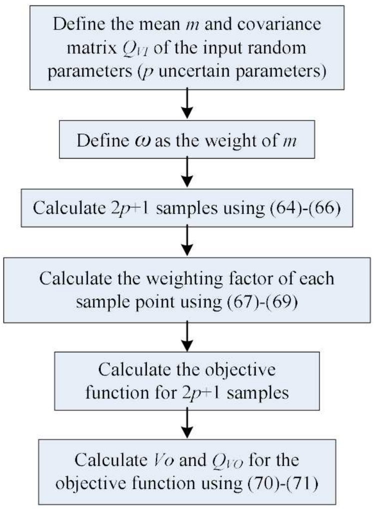

Fig. 3 *Flowchart of the proposed stochastic framework*

5.5 Case IV: analysis of the test system with two congested lines (lines 5–4 and 5–6)

In this case, it is assumed that the capacities of lines 5–6 and 5–4, which are connected to the EH bus, are limited, and it seems that these power lines have significant impact on EH performance.

Assume a day-ahead wholesale electricity market with several market players considering generators, loads and a strategic EH that aims to maximise its profit, in which all of them will submit their energy/price bids.

The optimal bidding process of pool-based electricity market actors is of two major types: economic bidding and self-schedule bidding. A market participant uses self-schedule bidding when it tries to submit an energy bid to the market without considering its price, while a market participant uses economic bidding when it tries to submit an energy bid considering its relative price [24]. In this paper, it has been assumed that the generators and the EH submit economic bids, whereas the demand bids are self-schedule. Table 1 shows the output generations of the proposed generators in all four cases.

Also, sum of the generated power of the producers and their deviation per cent from the base case are depicted in Table 1.

In case II, the generation power of the generator 1 is decreased about 22.72% due to the capacity limitation of line 4–5 and high price values of the power producer 1 compared with the others. As it is mentioned before, the capacity of line 5–6 is limited in case III. Hence, the generation power of the generator 1 is increased about 22.72% compared with the case I (same as the base case). The power generation of producers 2 and 3 is decreased about 8.93 and 19.58% due to the capacity limitation of power line 5–6. It can be said that the ISO tries to buy energy from the power generator 1 as a compromised option in terms of economic and load supplement. Since in case IV the capacities of transmission lines 4–5 and 5–6 are considered to be limited, the ISO would no longer buy energy from the generators 1 and 3. Hence, it tries to consume energy from generator 2. Table 1 shows that the generation power of producer 2 is increased about 18.47%.

Tables 2–4 shows the performance characteristics and coefficient of the EH and the input data of the water supplement system and the EH. Table 5 shows the average power of the EH equipment over the proposed four case studies as numerical values. It also shows the detailed performance of the EH in the proposed electricity market. The input powers of the EH at t = 00:00 to 12:00 AM are rarely low compared with the other hours of the day. However, an increasing trend in the early hours of the day can be seen due to the increasing rate of the price bid of the market participants. Simply, if we divide the day-ahead hours into two

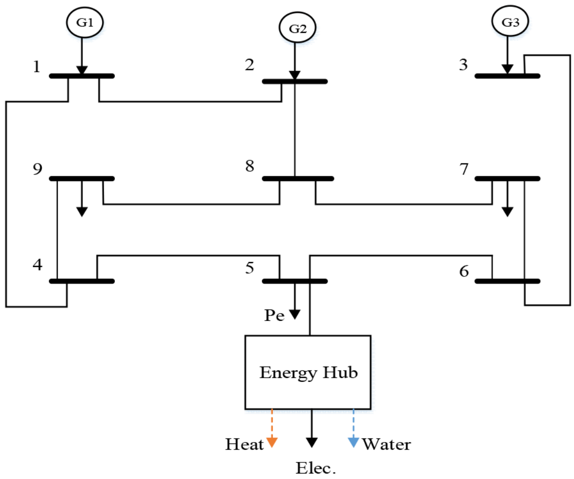

Fig. 4 *Schematic diagram of the IEEE test system*

Table 1 Comparison of generation units output

Time, h	Case I			Case II			Case III			Case IV		
	Pg1	Pg2	Pg3	Pg1	Pg2	Pg3	Pg1	Pg2	Pg3	Pg1	Pg2	Pg3
1	10	10	40	10	10	40	10	19.5	40	10	19.5	40
2	10	65	21.3	10	56.4	30	10	65	11.8	10	65	11.8
3	10	10	50	10	10	50	10	10	49.4	10	10	49.4
4	10	10	55	10	10	55	10	10	50.5	10	71.2	10
5	10	80	23.7	10	80	37.8	10	80	10	10	80	10
6	10	10	65	10	10	65	10	10	52.8	10	10	52.8
7	10	10	70	10	22.3	70	10	10	54	10	10	54
8	10	10	45.4	10	10	75	10	10	55.1	10	28.4	42.9
9	10	100	10	10	100	10	10	100	10	10	100	10
10	115	10	10	44.1	10	41.6	115	10	38.5	77.8	10	45.2
11	10	10	83.4	10	10	90	10	10	58.6	10	10	58.6
12	10	10	43.7	10	10	95	10	10	59.7	10	10	59.7
13	10	10	10	10	10	43.3	10	10	50	10	10	60.9
14	135	10	10	135	10	10	135	10	10	135	10	10
15	10	10	10	10	10	10	10	10	10	10	10	10
16	10	10	10	10	10	10	10	10	10	10	10	10
17	150	10	10	150	10	10	150	10	10	150	10	10
18	10	145	10	10	145	10	10	145	10	10	145	10
19	10	10	10	10	10	10	10	10	10	10	10	10
20	165	10	10	67.8	10	10	165	10	10	165	10	10
21	10	10	10	10	10	10	10	10	10	10	10	10
22	10	165	10	10	165	10	10	165	10	10	165	10
23	180	10	10	175	10	10	180	10	10	180	10	10
24	10	18.5	10	10	100.1	10	10	18.5	10	10	100.1	10
sum	935	753.5	637.5	761.9	838.8	812.7	935	763	650.4	897.8	924.2	615.3
deviation, %	—	—	—	−22.72	10.17	21.56	0	1.24	1.98	−4.14	18.47	−3.61

Table 2 Performance characteristics of the EH

η_{gh}^{B}	η_{ee}^{T}	η_{e}^{ST}	η_{e}^{SO}	η_{ge}^{CHP}	η_{gh}^{CHP}
70%	98%	90%	90%	40%	45%

Table 3 Input data of the desalination unit and tanks

V_{min}^{T}, m3	V_{max}^{T}, m3	V_{max}^{DT}, m3	W_{min}^{ID}, m3	W_{max}^{ID}, m3	CF^{Des}, MW/l
0	100	100	5	50	4

Table 4 Boundary values of the eh parameters

$\bar{P}_{eh}$, MW	$\bar{P}^{GI}$, MW	$\bar{P}^{B}$, MW	$\bar{P}^{T}$, MW	$\bar{P}^{CHP}$, MW	$\bar{S}^{re}$, MW
500	300	200	100	100	100

Table 5 Average optimal values of the variables of the proposed case studies

Time, h	EH's CHP input power, MW	EH's boiler input power, MW	EH's trans power exchange, MW
1	22.2	0	−5
2	26.7	0	−38.3
3	31.1	0	−9
4	35.6	0	−11
5	40	0	−46.7
6	44.4	0	−15
7	48.9	0	−17
8	53.3	0	10.6
9	57.8	0	−41
10	57.8	0	−53
11	80	0	−18.4
12	71.1	0	24.3
13	177.8	0	61
14	250	31.3	−46.9
15	250	34	102
16	250	36.7	73.5
17	250	39.3	38.8
18	250	50	−55.1
19	250	10	79
20	250	23.3	8.8
21	250	10	102
22	222.2	0	14.2
23	177.8	0	−74.4
24	124.4	0	85.5

main parts, which are 'low prices hours' and 'high price hours', it shows that the EH, as a strategic producer, tries to consume (buy) power from the power grid or other external sources (e.g. gas or water) in 'low prices hours' and inject (sell) power in 'high prices hours' in order to eliminate its thermal–electrical–water demands and also maximise its profit. The EH transformer values explicitly show that the EH tries to consume power during hours with low price and inject power during high price hours. Fig. 5 shows the LMPs of the network buses with and without connection of the EH to the power network. As mentioned above, the price-maker producer can alter the market prices to its profit. Fig. 5*b* clearly shows that the EH has changed the LMPs to maximise its profit. It also illustrates that the connection of the EH to the power grid has led the LMPs being deviated from their basic values whether they may be increased or decreased for some hours. It also shows that the LMPs are the same when no lines congested.

Fig. 6 shows the congestion addressing of the power lines. It shows that the impact of the congestion of the power lines on the EH profit is not the same. In this case, we have gradually decreased the capacity of all the power lines from 150 to 30 MW, independently. One can see that lower capacities of lines 1 and 4 result in more profit for the EH, which means that these power lines have positive impact on the EH performance. While the lower capacities of lines 7 and 9 lead to lower profit, which shows that the proposed lines congestion could have negative impact on the EH performance. Fig. 6 also shows the infeasible region of line 6, which means that the proposed ED would no longer be infeasible, if the capacity of line 6 gets the value below 60 MW. Fig. 7 illustrates the EH water exchange with grid. The EH and the battery power exchange with grid are shown in Figs. 8 and 9, respectively. It should be mentioned that the positive values indicate the generated power and negative values indicate the consumption power. It is noteworthy to say that the selling/buying water to/from the grid depends on the electricity network status. On the other hand, during the congestion scenario when the battery is not able to be charged, and it is not available to consume electricity from the power grid, the EH will serve its demand by buying water from the grid instead of using the desalination unit. Without loss of

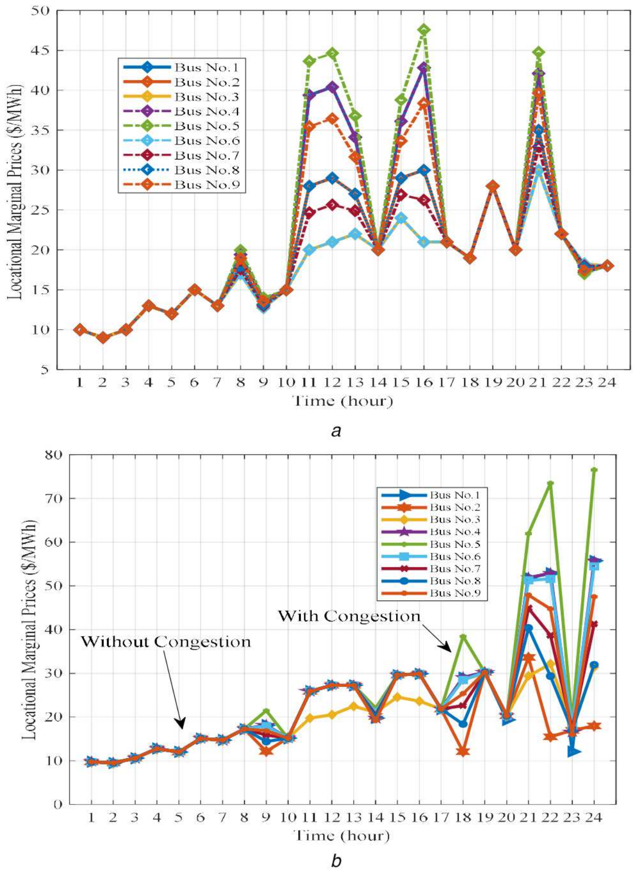

Fig. 5 *LMP of the network buses*
(a) Without EH, ***(b)*** With EH

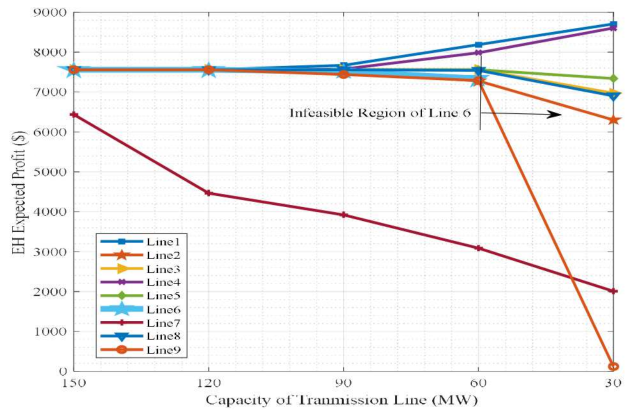

Fig. 6 *Congestion addressing of the power lines*

generality, it can be said that the congestion occurrence limits the EH performance. Fig. 9 represents charging/discharging power of the EH's battery unit in the proposed four cases. According to the economic bidding of the EH, the battery energy storage of the EH prefers to be charged during off-peak hours while discharging energy to the power grid during peak hours.

5.6 EH's profit

Performance analysis of the EH is carried out by considering four different case studies which have been clarified in previous sections. Fig. 10 represents the EH's profit in the proposed case studies. Fig. 10 shows that EH consumes lower profit in cases II and III compared with the base case. However, the congestion of both adjacent lines connected to the EH results in gaining the EH profit. Also, it must be mentioned that different lines' impact on the profit may differ depending on the connection point of the lines.

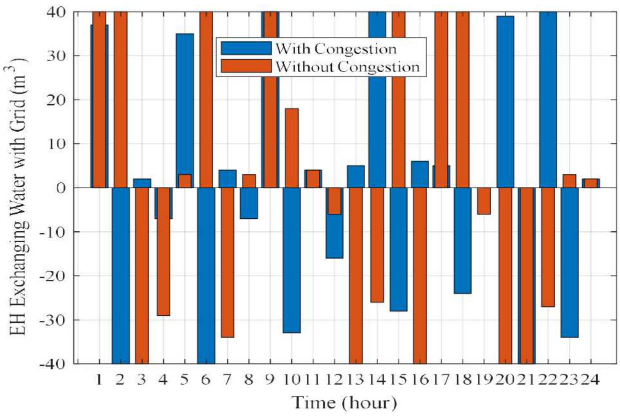

Fig. 7 *EH water exchange with grid*

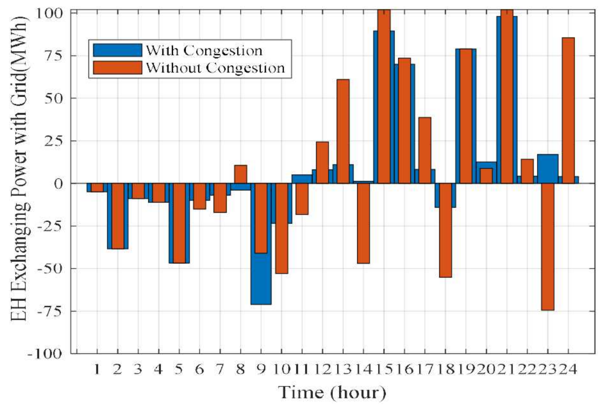

Fig. 8 *EH power exchange with grid*

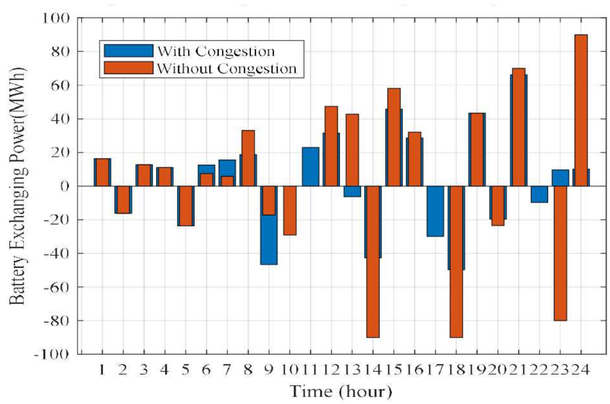

Fig. 9 *Battery power exchange with grid*

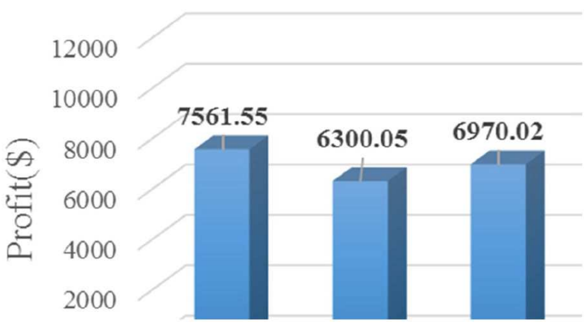

Fig. 10 *Comparison of the profit of the proposed case studies*

6 Conclusion

This paper proposes the operation of a large-scale EH in a transmission-constraint day-ahead electricity market. Thermal–water–electrical demands of the EH have been considered in this paper. The proposed EH considered as an integrated EH which all its generation units belong to one particular independent owner. The proposed stochastic framework based on UT could model the high uncertainties of the EH in the pool-based market, properly. The initial problem was formulated based on MPEC which is non-linear and need to be dealt with. Then, the problem is transformed into an MILP which is more preferred and leads to more precise answers. Different case studies assumed in this work in terms of congestion concept to assess performance behaviour of the EH in the presence of congested lines. One can conclude that some time congestion provides the opportunity of making more profit for the proposed producer. The impact of congested line on profit would be different among all power grid lines depending on the connection points of the lines. Results effectively show the congestion oriented and overall performance of the EH in the proposed electricity market. The proposed water energy thermal model can provide a very appropriate way of analysing the modern EH, considering different types of demands. The simulation results advocate the main merits of the proposed model.

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